Exercise 3B: Solutions

1 **a**
$$\begin{aligned} x^2+2x &= -1 \\ x^2+2x+1 &= 0 \\ (x+1)^2 &= 0 \\ x &= -1 \end{aligned}$$

b
$$\begin{aligned} x^2-6x+9 &= 0 \\ (x-3)^2 &= 0 \\ x &= 3 \end{aligned}$$

c Divide both sides by 5:
$$\begin{aligned} x^2-2x &= \frac{1}{5} \\ x^2-2x+1 &= \frac{6}{5} \\ (x-1)^2 &= \frac{6}{5} = \frac{30}{25} \\ x-1 &= \pm\frac{\sqrt{30}}{5} \\ x &= 1 \pm \frac{\sqrt{30}}{5} \end{aligned}$$

d Divide both sides by -2:
$$\begin{aligned} x^2-2x &= -\frac{1}{2} \\ x^2-2x+1 &= \frac{1}{2} \\ (x-1)^2 &= \frac{1}{2} = \frac{2}{4} \\ x-1 &= \pm\frac{\sqrt{2}}{2} \\ x &= 1 \pm \frac{\sqrt{2}}{2} \end{aligned}$$

e Divide both sides by 2:
$$\begin{aligned} x^2+2x &= \frac{7}{2} \\ x^2+2x+1 &= \frac{9}{2} \\ (x+1)^2 &= \frac{9}{2} = \frac{9\times 2}{4} \\ x+1 &= \pm\frac{3\sqrt{2}}{2} \\ x &= -1 \pm \frac{3\sqrt{2}}{2} \end{aligned}$$

f
$$\begin{aligned} 6x^2+13x+1 &= 0 \\ x &= \frac{-13\pm\sqrt{169-4\times 6\times 1}}{12} \\ &= \frac{-13\pm\sqrt{145}}{12} \end{aligned}$$

2 a $\Delta = 9 - 4m$

No solutions: $\Delta < 0$

$9 - 4m < 0$

$m > \frac{9}{4}$

b $\Delta = 25 - 4m$

Two solutions: $\Delta > 0$

$25 - 4m > 0$

$m < \frac{25}{4}$

c $\Delta = 25 + 32m$

One solution: $\Delta = 0$

$25 + 32m = 0$

$m = -\frac{25}{32}$

d $\Delta = m^2 - 36$

Two solutions: $\Delta > 0$

$m^2 - 36 > 0$

$m > 6 \text{ or } m < -6$

e $\Delta = m^2 - 16$

Nosolutions: $\Delta < 0$

$m^2 - 16 < 0$

$-4 < m < 4$

f $\Delta = m^2 + 16m$

One solution: $\Delta = 0$

$m^2 + 16m = 0$

$m = -16 \text{ or } m = 0$

3 a $2x^2 - x - 4t = 0$

$$x = \frac{1 \pm \sqrt{1 - 4 \times 2 \times -4t}}{4}$$

$$= \frac{1 \pm \sqrt{32t + 1}}{4}$$

$32t + 1 \geq 0$

$32t \geq -1$

$t \geq -\frac{1}{32}$

b $4x^2 + 4x - t - 2 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -(t+2)}}{8}$$

$$= \frac{-4 \pm \sqrt{16 + 32 + 16t}}{8}$$

$$= \frac{-4 \pm \sqrt{16t + 48}}{8}$$

$$= \frac{-4 \pm 4\sqrt{t + 3}}{8}$$

$$= \frac{-1 \pm \sqrt{t + 3}}{2}$$

$t + 3 \geq 0$

$t \geq -3$

c $5x^2+4x-t+10=0$

$$\begin{aligned} x&=\frac{-4\pm\sqrt{16-4\times5\times(-t+10)}}{10}\\ &=\frac{-4\pm\sqrt{16+20t-200}}{10}\\ &=\frac{-4\pm\sqrt{20t-184}}{10}\\ &=\frac{-4\pm\sqrt{4(5t-46)}}{10}\\ &=\frac{-4\pm2\sqrt{5t-46}}{10}\\ &=\frac{-2\pm\sqrt{5t-46}}{5}\end{aligned}$$

$$\begin{aligned} 5t-46&\geq0\\ 5t&\geq46\\ t&\geq\frac{46}{5}\end{aligned}$$

d $tx^2+4tx-t+10=0$

$$\begin{aligned} x&=\frac{-4t\pm\sqrt{16t^2-4\times t\times(-t+10)}}{2t}\\ &=\frac{-4t\pm\sqrt{16t^2+4t^2-40t}}{2t}\\ &=\frac{-4t\pm\sqrt{20t^2-40t}}{2t}\\ &=\frac{-4t\pm2\sqrt{5t^2-10t}}{2t}\\ &=\frac{-2t\pm\sqrt{5t(t-2)}}{t}\end{aligned}$$

$5t(t-2)\geq0$

This is a quadratic with a minimum and solutions $t=0, t=5$.

$\therefore \quad t<0,\ t\geq2$

Note: $t=0$ gives denominator zero, so it must be checked by substituting $t=0$ in the original equation. In this case it gives $10=0$, and so is not a solution, but it should be checked.

(e.g. $tx^2+5x+4=t$ gives a solution with t on the denominator, but substituting $t=0$ gives $5x+4=0$, which has a solution.)

4 **a**
$$\begin{aligned} x&=\frac{-p\pm\sqrt{p^2-4\times1(-16)}}{2}\\ &=\frac{-p\pm\sqrt{p^2+64}}{2}\end{aligned}$$

b $p=0$ gives $x=\dfrac{0+\sqrt{64}}{2}=4$

$p=6$ gives $x=\dfrac{-6+\sqrt{100}}{2}=2$

5 **a** $2x^2-3px+(3p-2)=0$

$$\begin{aligned} \Delta&=9p^2-8(3p-2)\\ &=9p^2-24p+16\\ &=(3p-4)^2\end{aligned}$$

$\Delta$is a perfect square

b $\Delta=0\Rightarrow p=\dfrac{4}{3}$

c Solution is $x = \dfrac{3p \pm (3p-4)}{4}$

i When $p = 1, x = \dfrac{3 \pm 1}{4}$

$\therefore x = 1$ or $x = \dfrac{1}{2}$

ii When $p = 2, x = \dfrac{6 \pm 2}{4}$

$\therefore x = 2$ or $x = 1$

iii When $p = -1, x = \dfrac{-3 \pm 7}{4}$

$\therefore x = 1$ or $x = -\dfrac{5}{2}$

6 **a** $4(4p-3)x^2 - 8px + 3 = 0$

$$\begin{aligned}\Delta &= 64p^2 - 48(4p-3)\\ &= 64p^2 - 192p + 144\\ &= 16(4p^2 - 12p + 9)\\ &= 16(2p-3)^2\end{aligned}$$

Δ is a perfect square

b $\Delta = 0 \Rightarrow p = \dfrac{3}{2}$

c Solution is $x = \dfrac{8p \pm 4(2p-3)}{8(4p-3)}$

That is $x = \dfrac{1}{2}$ or $x = \dfrac{3}{2(4p-3)}$

i When $p = 1, x = \dfrac{8 \pm 4}{8}$

$\therefore x = \dfrac{1}{2}$ or $x = \dfrac{3}{2}$

ii When $p = 2, x = \dfrac{16 \pm 20}{40}$

$\therefore x = \dfrac{1}{2}$ or $x = \dfrac{3}{10}$

iii When $p = -1, x = \dfrac{-8 \pm 20}{-56}$

$\therefore x = \dfrac{1}{2}$ or $x = -\dfrac{3}{14}$

7 Use Pythagoras' Theorem:

$$\begin{aligned}(8-x)^2 + (6+x)^2 &= 100\\ 64 - 16x + x^2 + 36 + 12x + x^2 &= 100\\ 2x^2 - 4x &= 0\\ 2x(x-4) &= 0\\ x &= 2 \text{ since } x \neq 0\end{aligned}$$

8 Let x be the length of one part.
The other part has length $100 - x$
Let the second one be the larger.

$$\begin{aligned}\left(\frac{200-x}{4}\right)^2 &= 9\frac{x^2}{16}\\ (200-x)^2 &= 9x^2\\ 200 - x &= 3x\end{aligned}$$

$x = 50$

$\therefore 200 - x = 150$

The length of the sides of the larger square is 37.5 cm

9 **a** Let $a = \sqrt{x}$

$a^2 - 8a + 12 = 0$

$(a-6)(a-2) = 0$

$a = 6 \text{ or } a = 2$

$\therefore x = 36 \text{ or } x = 4$

b Let $a = \sqrt{x}$

$a^2 - 2a - 8 = 0$

$(a-4)(a+2) = 0$

$a = 4 \text{ or } a = -2$

$\therefore x = 16$

c Let $a = \sqrt{x}$

$a^2 - 5a - 14 = 0$

$(a-7)(a+2) = 0$

$a = 7 \text{ or } a = -2$

$\therefore x = 49$

d Let $a = \sqrt[3]{x}$

$a^2 - 9a + 8 = 0$

$(a-8)(a-1) = 0$

$a = 8 \text{ or } a = 1$

$\therefore x = 512 \text{ or } x = 1$

e Let $a = \sqrt[3]{x}$

$a^2 - a - 6 = 0$

$(a-3)(a+2) = 0$

$a = 3 \text{ or } a = -2$

$\therefore x = 27 \text{ or } x = -8$

f Let $a = \sqrt{x}$

$a^2 - 29a + 100 = 0$

$(a-25)(a-4) = 0$

$a = 25 \text{ or } a = 4$

$\therefore x = 625 \text{ or } x = 16$

10 $3x^2 - 5x + 1 = a(x^2 + 2bx + b^2) + c$ Equating coefficients:

$x^2 : \quad 3 = a$

$x : \quad -5 = 2ba \Rightarrow b = -\frac{5}{6}$

constant: $\quad 1 = b^2 a + c \Rightarrow c = -\frac{13}{12}$

Minimum value is $-\frac{13}{12}$

11 $2 - 4x - x^2 = 24 + 8x + x^2$

$2x^2 + 12x + 22 = 0$

$x^2 + 6x + 11 = 0$

$\Delta = 36 - 4 \times 11 < 0$

Therefore no intersection

12 $(b-c)x^2+(c-a)x+(a-b)=0$

$$((b-c)x-(a-b))(x-1))=0$$

$$x=\frac{a-b}{b-c} \text{ or } x=1$$

13 $2x^2-6x-m=0$

$$x=\frac{6\pm\sqrt{36+8m}}{4}$$

The difference of the two solutions

$$=\frac{\sqrt{36+8m}}{2}$$

$$\frac{\sqrt{36+8m}}{2}=5$$

$$36+8m=100$$

$$8m=64$$

$$m=8$$

14a $(b^2-2ac)x^2+4(a+c)x-8=0$

$$\begin{aligned}\Delta &= 16(a+c)^2+32(b^2-2ac)\\ &= 16(a^2+2ac+c^2)+32b^2-64ac\\ &= 16a^2-32ac+16c^2+32b^2\\ &= 16(a^2-2ac+c^2+2b^2)\\ &= 16((a-c)^2+2b^2)>0\end{aligned}$$

b One solution if $a=c$ and $b=0$

15 $\frac{1}{2}+\frac{1}{x+k}=\frac{1}{x}$

$$x(x+k)+2x=2(x+k)$$

$$x^2+xk+2x=2x+2k$$

$$x^2+kx-2k=0$$

$$\Delta=k^2+8k$$

$$\Delta<0\Rightarrow k^2+8k<0$$

$$k^2+8k<0$$

$$k(k+8)<0$$

$$-8<k<0$$

16 $3x^2+px+7=0\ \Delta=p^2-84$

$p^2>84$

The smallest such integer is 10.